Matching Models of Markets

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Summary

This dissertation studies the existence and structure of stable outcomes in several matching models which generalize and extend the classic frameworks of Gale–Shapley (1962) and Shapley–Shubik (1971), as well as the more recent work of Gul–Stacchetti (1999, 2000), Hatfield–Milgrom (2005), Sun–Yang (2006, 2009), and Ostrovsky (2008). Several distinct contract structures are considered, in an attempt to identify and expand the limits and possibilities of centralized design for promoting stability in matching markets.

First, a general "matching in networks" model, which embeds previous two-sided and supply chain matching models, is considered. In this model, agents trade (indivisible) goods via bilateral contracts. Studying the general setting of matching in networks elucidates that (1) acyclicity of the contract set and (2) full substitutability of agents' preferences—two assumptions which have been maintained throughout the previous literature—are in fact necessary for the guaranteed existence of stable outcomes. Meanwhile, if both of these assumptions are satisfied, then stable outcomes can always be found using a suitable generalization of the deferred acceptance algorithm. Moreover, in this case the classical lattice structure, rural hospitals theorem, and one-sided strategy-proofness results all extend.

In the setting of matching in networks, outcomes are affected by the way in which contractual primitives such as goods are bundled into contracts for exchange. This observation is more simply illustrated in the context of many-to-many matching with contracts: If every contractual relationship of every pair of agents must be expressed through a single contract, then such models directly correspond to a form of wage bargaining. By contrast, when pairs of agents are allowed to express their relationships through multiple contracts, this correspondence no longer holds and a tradeoff arises—increased bundling makes the contractual language less expressive, but increases the likelihood that agents will exhibit substitutable preferences over contracts.

Next, it is shown that when continuously transferable utility is included in (bilateral) contract matching markets and agents' utilities are quasilinear in the numeraire, acyclicity is unnecessary—full substitutability of preferences is on its own sufficient to guarantee the existence of stable outcomes for any underlying network structure; such outcomes can be computed via reduction to the Kelso–Crawford (1982) cumulative offer process. Furthermore, in this case analogues of the first and second welfare theorems hold, the set of stable outcomes is essentially equivalent to the set of competitive equilibria, and all stable outcomes are in the core and are efficient. In contrast, for any domain of preferences strictly larger than that of full substitutability, neither competitive equilibria nor stable outcomes can be guaranteed to exist.

Finally, if contracts are allowed to be continuously divisible (and continuous transfers are available), as in the case of contracting over joint research venture participation, then "multilateral contracts," which embed some production complementarities previously outside the scope of matching theory, may be studied within the matching framework. As in the case of discrete contracts, analogues of the first and second welfare theorems hold. If agents' utilities are concave in venture participation, then competitive equilibria exist, can be computed efficiently, correspond to stable outcomes, and yield core outcomes.

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Publication Credits

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John William Hatfield and Scott Duke Kominers. Matching in networks with bilateral contracts (extended abstract). In *Proceedings of the 11th ACM Conference on Electronic Commerce*, 2010.